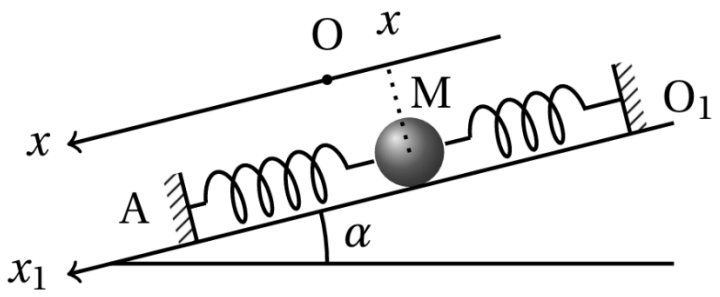
Exercise Set 8 (TD8)

Exercise 1: Double spring on an inclined plane

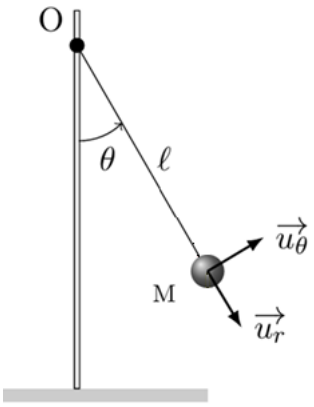
A ball, considered as a material point M with mass m, is attached to two identical springs (with stiffness k and natural length ℓ_0) whose ends are fixed at points O_1 and A ($O_1A = \ell_0$) on a plane inclined at an angle α to the horizontal plane (see figure). The ball slides without friction along the inclined plane on the axis O_1x_1.



1. Add on the diagram the forces exerted on the mass m.
2. Determine the x-coordinate $x_e = O_1M_e$ of the material point in equilibrium.
3. We place the origin O at point M_e and we set $x_1 - x_e = x$, with x the x-coordinate of point M with respect to the point of equilibrium. Determine the second-order differential equation for the $x(t)$ (we will use a natural frequency ω_0), and then solve this equation, knowing that the material point is initially at the point of equilibrium with initial velocity $\vec{v}_0 = v_0 \vec{u}_x$.

Exercise 2: Studying energy in the simple pendulum

A mass m is suspended from an inextensible string of length ℓ. Its motion can be described by the angle θ that the string makes with the vertical. Friction is neglected.



1. Give the general expression for the gravitational potential energy of the system as a function of m, g, ℓ, and $\dot{\theta}$. When is it at its maximum? When is it at its minimum?
2. Give the general expression for the kinetic energy of the system as a function of m, ℓ, and θ. When is it at its maximum? When is it at its minimum?
3. Give the expression for the mechanical energy. What can be said about the mechanical energy in this specific case?
4. Use the conservation of mechanical energy to find the equation of a harmonic oscillator.
5. The mass is released while it is at rest and $\theta(0) = \theta_0$. Given these initial conditions, determine the evolution of the angle θ over time. Express it as a function of θ, ℓ, and g.
6. Plot the curve giving θ as a function of time between $[2\pi; 2\pi]$. Do the same for kinetic, gravitational potential, and mechanical energies.