Exercise Set 6 (TD6)

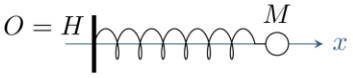
Exercise 1: Free-body diagrams in springs

In each case below, write the force exerted by the spring on the mass fixed at point M in terms of the stiffness k and the natural length ℓ_0 of the spring, the position x or z of point M, the position x_H or z_H of point H where the spring is fixed, and the unit vector $\vec{u}_x$ or $\vec{u}_z$.

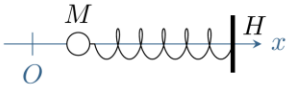
The origin is always at point O.

In the last case, write the forces exerted by the two springs on points M_1 and M_2, with x-coordinates x_1 and x_2 respectively. The two springs are different and are described by the quantities k, ℓ_0 and k', ℓ'_0.

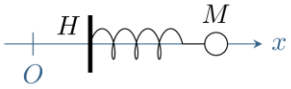
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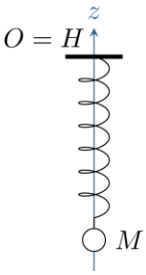
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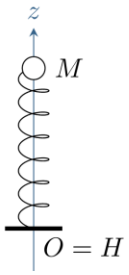
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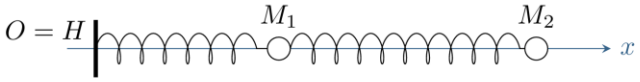
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5 -

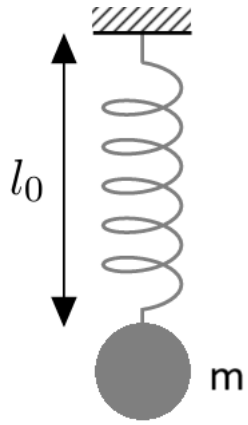


6 -



Exercise 2: Vertical spring

A ball of mass m is suspended from a vertical spring with stiffness constant k and unstretched length l_0. The mass is released at an initial length of the spring $L = 3l_0/2$ with zero initial velocity. We assume that friction is negligeable.

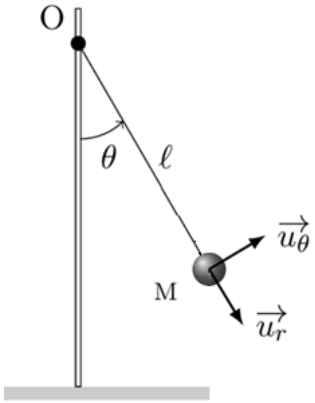


1. Draw the forces exerted on the mass on the diagram.
2. Use Newton's second law to find the spring's position of equilibrium.
3. Use Newton's second law to find an equation for the acceleration vector.
4. Project your expression on the axis that is relevant to this problem.
5. Write the differential equation in its canonical form.
6. Solve the differential equation.

Exercise Set 7 (TD7)

Exercice 1: Simple pendulum

A ball of mass M is suspended from an inextensible string of length l. The ball is released at an angle θ_0 with the vertical axis with zero angular velocity $\dot{\theta}$.



1. Draw on the diagram the forces that are exerted on the mass..
2. Use Newton's second law to find the equation for the acceleration vector.
3. Project your expression on the axes shown on the diagram.
4. Write the differential equation in its canonical form.
5. Solve the differential equation using the initial conditions given above.

Remember that the general expression for the acceleration vector in polar coordinates is

$$\overrightarrow{a(t)} = \left(\ddot{r} - r\dot{\theta}^2\right)\overrightarrow{u_r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\overrightarrow{u_\theta}$$

Exercice 2 : Vibrational mode of a molecule of HCl

The vibrational mode of the molecule of hydrogen chloride (HCl) is spectroscopically measured to be $f = 8.5 \cdot 10^{13}$ Hz. Here we will consider a simple model of this molecule, by describing it as a mobile hydrogen atom connected to a fixed chlorine atom. The interaction between the two atoms is modeled by a spring of stiffness k.

Data given: $M_H = 1.0\ \text{g/mol}$; $M_{Cl} = 35.5\ \text{g/mol}$; Avogadro's number $N_A = 6.0 \cdot 10^{23}\ \text{mol}^{-1}$.

1. Why is it reasonable to assume that the chlorine atom is fixed?

It is given that the total energy of the molecule is $E_{total} = \frac{1}{2}hf$ where $h = 6.62 \cdot 10^{-34}\ \text{J·s}$ is a constant (known as Planck's constant) and f is the frequency of the vibrational mode in Hertz. Frequency is connected with angular frequency with the formula: $f = \frac{\omega_0}{2\pi}$.

2. Calculate the spring's constant of stiffness k.
3. At which moment is the kinetic energy of the molecule at its maximum?
4. Give the connection between the total energy the kinetic energy at that moment. Use it to calculate the maximum velocity of the hydrogen atom.
5. Qt what moment is the elastic potential energy of the molecule at its maximum?
6. Give the connection between the total energy and the elastic potential energy at that moment. Use it to calculate the amplitude of the hydrogen atom's motion. Compare your result with the length of the H-Cl bond that is 127pm.