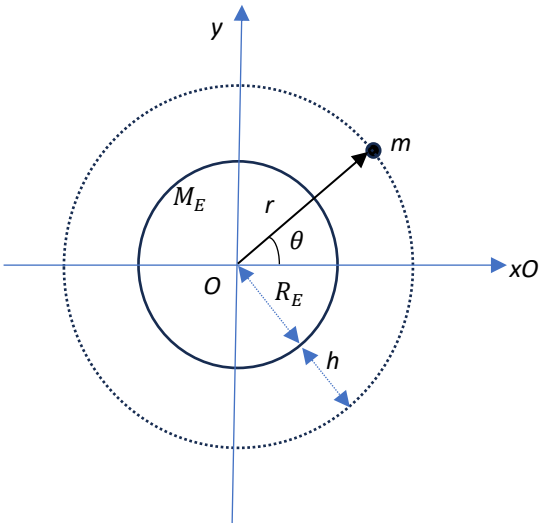
Exercise Set 3 (TD3) - Mechanics

Dynamics: Newton's laws

Exercise 3: Motion of a satellite

A satellite with mass m = 300 kg is moving on a circular orbit around the Earth at a height h above the Earth's surface, equal to its radius h = R_E = 6400 km. Assume that at time t it is located at point M with polar coordinates (r,θ) as shown in figure. The origin of the axes is located at the center of the Earth.



1. Write the position vector $\overrightarrow{OM}$ in polar coordinates.
2. Using the fact that the radius $r = R_E + h$ is constant, write the velocity vector $\vec{v}$, as well as its norm, in polar coordinates.
3. Write the acceleration vector $\vec{a}$ with the same assumption.
4. Write the vector for the gravitational force $\vec{F}$ exerted on the satellite.
5. Using Newton's second law, determine the acceleration vector.
6. Using the equations established above, show that $\dot{\theta} = \omega$ is constant and then show that the velocity vector can be written as $\|\vec{v}\| = v = r\omega$.
7. Show that the norm of the acceleration can be written as $\|\vec{a}\| = a = \frac{v^2}{r}$.
8. Using the above results, determine v as a function of G, M_E, and r. Calculate its numerical value using $G = 6.67 \cdot 10^{-11}$ Nm^2/kg^2 and $M_E = 6 \cdot 10^{24}$ kg.
9. Determine the period T of the satellite, that is the time it takes for a full rotation around the Earth. Calculate its numerical value in hours.