Exercise Set 3 (TD3) - Mechanics

Dynamics: Newton's laws

Exercise 1 : Free Fall

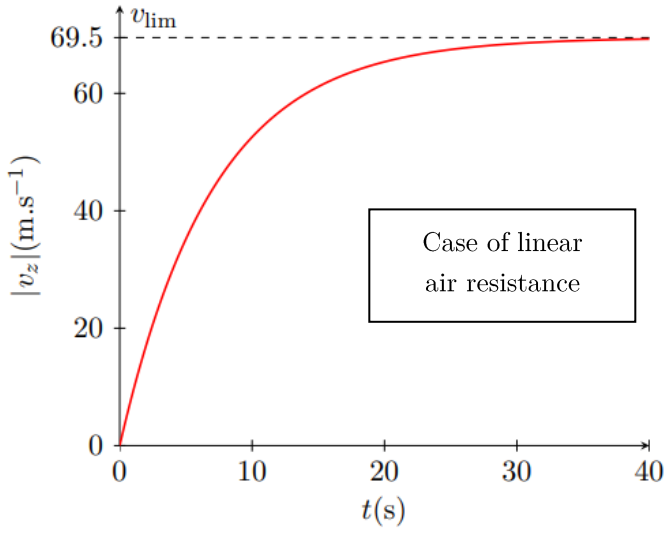
We study the free fall of a parachutist of mass $m = 80\ \text{kg}$ who jumps from a height of $y(0) = 3000\ \text{m}$ with zero initial velocity.

a. Case without air resistance

1. Describe the study framework (reference frame, system under study, coordinate system).
2. Using Newton's second law, determine the acceleration vector.
3. Determine the velocity vector.
4. Determine the position vector.
5. Calculate the duration of the fall.
6. What is the velocity of the parachutist when it reaches the ground? Comment on your result.

b. Case with air resistance

7. Using Newton's second law, determine the acceleration vector.
8. Show that the obtained equation can be rewritten as
$$\frac{dv}{dt} + \frac{v}{\tau} = -g$$
What is τ ? How is this equation called?
9. Find the solution $v(t)$.
10. The norm of the velocity vector depends on time according to the following function
$$|v(t)| = g\tau\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$
and is plotted in the graph below.
What is the maximum speed of the parachutist in this case? Compare to your answer to question 6.



Exercise 2: Penalty in rugby

«*In rugby, a player scores a goal from a penalty kick when he kicks the ball over the crossbar and between the opposing team's goal posts.*" (Extract from rule 2, "Game rules", FFR - French Rugby Federation).

In this exercise, we will consider the rugby ball to be a material point centered at point M with mass m = 420g. The origin (O, x, y) is placed at the ball's position right before the kick.



The ball is kicked by the player with an initial velocity $\vec{v}_0$, with norm $\|\vec{v}_0\| = 23\ \mathrm{m{\cdot}s^{-1}}$ at an angle $\beta = 38°$ with the ground. We assume that the player is perfectly aligned with the opposing team's goal posts at a distance $d = 40\ \mathrm{m}$ from them. The height of the crossbar is 3 m, while the height of the vertical posts is 8 m. We ignore here any resistance on the ball's motion from the air.

1. Describe the study framework (reference frame, system under study, coordinate system).
2. Using Newton's second law, determine the acceleration vector.
3. Determine the velocity vector.
4. Determine the position vector.
5. Establish the equation for the trajectory $y(x)$. What type of trajectory is it?
6. Has the player scored the goal?