Exercise Set - TD 3: Thermodynamics

Exercise 1

Prove that it is impossible to create an engine with a single heat source. Use the two laws of thermodynamics to obtain a set of equations showing a contradiction, and therefore, proving that it is impossible for a single-source engine to exist.

Exercise 2

1. Draw the energy diagram of a heat pump.
2. Calculate its COP by explaining in detail each step.
3. What is its value when all processes are reversible?

Exercise 3

The Stirling engine consists of two chambers, a hot and a cold one, connected by a regenerator of constant volume, which can be made of braided copper wires. The gas, which is in a closed circuit, receives heat from a hot source (for example a combustion boiler) and gives heat to the cold source (for example the atmosphere).

The role of the regenerator, which is the basis of Robert Stirling's invention, is fundamental to obtaining good efficiency. In his original patent of 1816, Stirling explains that the hot gas enters the hot part of the regenerator and is gradually cooled down during its path to exit at the other end at a temperature almost identical to the temperature of the cold source. In the reverse trajectory, the gas is gradually heated. This technological trick allows for part of the heat exchanges to be internal to the engine. Here we will assume that the cycle is operating with $n = 40$ mmol of air, which we will consider as an ideal gas.

Initially, we neglect the regenerator: the two chambers are assumed to be one. The Stirling cycle can then be modeled by the succession of two isothermal and two isochore processes starting from a state 1 (with $P_1 = 1$ bar, T_1 =300 K) and described as follows:

- $1 \rightarrow 2$: reversible isothermal compression at $T_c = T_1$ up to state 2 where $V_2 = V_1/10$;
- $2 \rightarrow 3$: isochoric heating in contact with the hot source at $T_h = 600$ K up to state 3 with $T_3 = T_h$;
- $3 \rightarrow 4$: reversible isothermal expansion in contact with the hot source at T_h up to state 4 with volume $V_4 = V_1$;
- $4 \rightarrow 1$: isochoric cooling in contact with the cold source back to state 1.

1. Calculate the numerical values of P, V and T for each of the four states.
2. Calculate for each step the work and the heat transfer received by the gas. For an ideal gas, we give $c_v = nR/0.4$.
3. Comment on these results: do we really have an engine cycle?
4. Give the expression and calculate the value for the cycle's efficiency.

The invention of the regenerator by Stirling made it possible to considerably improve the efficiency of the engine described above. His idea is to ensure that the gas exchanges heat during the processes $2 \rightarrow 3$ and $4 \rightarrow 1$ not with thermostats but with a system, the regenerator, which does not exchange any energy other than with the gas during these processes $2 \rightarrow 3$ and $4 \rightarrow 1$.

5. Justify Stirling's idea.
6. What is the efficiency in this case? Can it be improved without changing the sources?