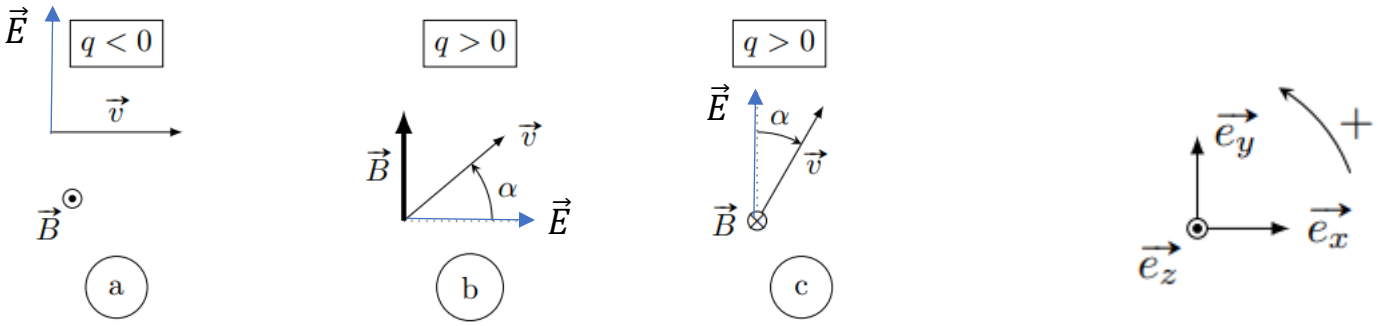
TD5: Electromagnetism

Charged particles moving in a magnetic field

Exercise 1: Lorentz force

Write the Lorentz force in each of the following cases:

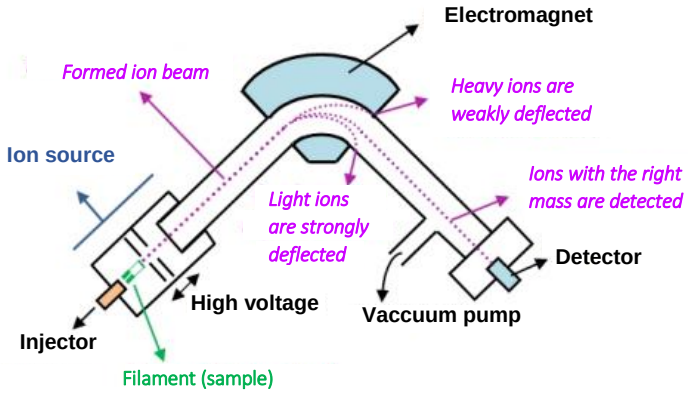


Exercise 2: Lorentz force

A charge $q = 3.2 \cdot 10^{-19}$ C is moving with velocity $v = (2\vec{u}_x + 3\vec{u}_y - \vec{u}_z)$ m/s in an area with a homogeneous electric and magnetic fields, $\vec{E} = (4\vec{u}_x - \vec{u}_y - 2\vec{u}_z)$ V/m and $\vec{B} = (2\vec{u}_x + 4\vec{u}_y + \vec{u}_z)$ T respectively. Calculate the Lorentz force that is exerted on the charge.

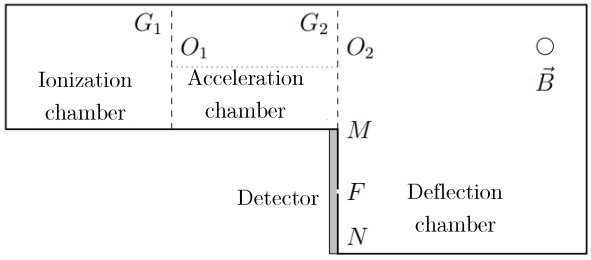
Exercise 3: Mass spectrometer

France produces most of its electricity – around 75% – from nuclear power plants. These plants use as an energy source a fuel consisting of uranium oxide enriched with around 4% uranium 235, the only fissionable isotope. Therefore, the ore must first be treated to produce this fuel before it can be used in a power plant. The purpose of uranium enrichment is to raise the ^{235}U content in the initial uranium to an optimal value for the desired application.



One of the methods used is mass spectrometry, which is the most sensitive method of isotopic analysis. A mass spectrometer consists of four parts:

- The ionization chamber, in which uranium atoms $^{235}_{92}U$ and $^{238}_{92}U$, with masses m_1 and m_2 respectively, are brought to high temperature and are ionized into ions U^+. We will assume that at the exit of this chamber at point O_1, the ions have practically zero velocity.
- The acceleration chamber, in which the ions are accelerated between points O_1 and O_2 under the action of a potential difference established between two grids G_1 and G_2.
- The deflection chamber in which the ions are deflected by a uniform magnetic field $\vec{B}$ with direction perpendicular to the plane of the figure.
- An ion collector is placed between points M and N. A slit centered at O_2 with width L in the plane of the figure makes it possible to choose the width of the incident beam. A collector slit centered at point F is placed between M and N and has width L_0, also in the plane of the figure.



The chambers are under vacuum, which allows us to assume that the particles do not interact with each other. We will neglect the weight of the ions in comparison with the electromagnetic force and we will assume that at the exit of the acceleration chamber, the velocities of the ions are lying in the plane of the figure.

1. What should be the direction of the field E_1 in the acceleration chamber? Justify your answer.
2. Give the explicit expression for the Lorentz force for a general case. Simplify it for the case in which the particle is in the acceleration chamber.
3. Establish the explicit expression for the velocity v of the ion when it reaches grid G_2. Remember that the kinetic energy is given by $E_c = \frac{1}{2}mv^2$ while the potential energy associated with the Lorentz force is $E_p = qV$.
4. The kinetic energy obtained by the ions at O_2 is 15.0 keV. Deduce the value of the potential difference U=$V_{G1} - V_{G2}$ applied between the two grids. Determine numerically the velocities v_{235} and v_{238} respecting the significant figures.
5. What must be the direction of the magnetic field $\vec{B}$ in the deflection chamber so that the ions can reach the collector?
6. Show that the motion in the magnetic field is uniform.
7. Using a method of your choice, determine the trajectory of a beam of $^{235}_{92}U^+$ ions in the area with the magnetic field (assuming that all ions in the beam have the same kinetic energy). Express the radius R_{235} of their trajectory as a function of m_{235}, e, U and $\vec{B}$. Do the same for beam of $^{238}_{92}U^+$ ions.
8. The collector, which consists of a metal container with a slit of width L_0 centered at F, between points M and N, collects the $^{235}_{92}U$ isotopes. What should be the value of the magnetic field in the spectrometer if the slit F is placed at a distance D = 940 mm from O_2?
9. The ion beam emitted at O_2 is a parallel beam in the plane of the figure. The collector slit has width L_0 = 4.0 mm in the plane of the figure. Can the isotopes be separated in the collector container?