Physics S1 (2025-2026)

Part 1: Classical Mechanics

I. Kinematics

Kinematics is a branch of mechanics in which we study motion independently of its cause. In this chapter, we will first define the tools necessary for the geometric description of the problem, and then we will study the physical quantities that are used to describe motion.

1. Definitions

Definition 1: Material point

A material point is a system whose spatial extension and rotation on itself can be both neglected. It therefore depends on the problem:

- The Earth, if we consider its motion around the Sun, has a negligible radius compared to the Earth-Sun distance: we can consider it a material point.
- If we consider the Earth's own rotational motion, to study the length of the day, for example, it obviously cannot be considered a material point: its spatial extension is an important parameter.

Definition 2: Reference frame

This is the space and time frame that we choose. For example, depending on the problem, we may choose a terrestrial, geocentric, or heliocentric reference frame. A reference frame that moves without any acceleration is called Galilean.

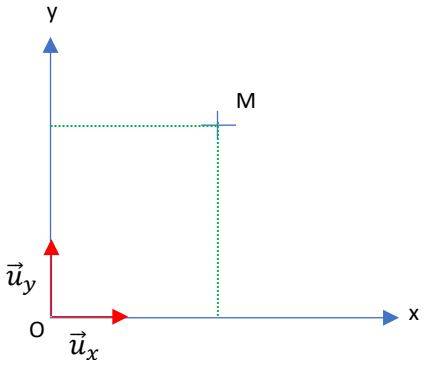
Definition 3: Coordinate system

To locate a point M in space, we use a coordinate system. There are several, each more or less adapted to a particular geometry.

2. Coordinate systems

a. Cartesian coordinates

In cartesian coordinates, we place point O at the crossing of the x and y axes. The reference frame is provided by two fixed unit vectors $\vec{u}_x$ and $\vec{u}_y$ so that $(\vec{u}_x, \vec{u}_y)$ forms a right-handed orthonormal basis. Point M is defined by two parameters x and y where



- x is the projection of point M on the axis $(O, \vec{u}_x)$
- y is the projection of point M on the axis $(O, \vec{u}_y)$

The position vector, that points to point M, is written as:

$$\overrightarrow{OM} = x \cdot \vec{u}_x + y \cdot \vec{u}_y = \begin{pmatrix} x \\ y \end{pmatrix}$$

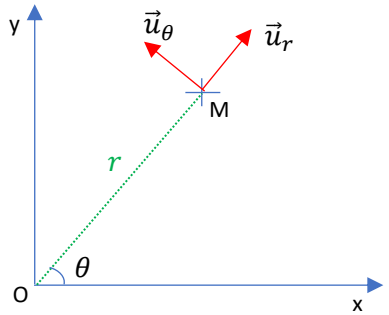
The cartesian coordinate system is suitable for studying most cases of motion in plane except for circular motion.

b. Polar coordinates

In polar coordinates, point O is also at the origin of the x and y axes, but point M is defined by two different parameters:

- The distance r between point M and the origin O. It is always positive since it is a distance.
- The angle θ between the vector $\overrightarrow{OM}$ and the axis $(O, \vec{u}_x)$.

We now define a right-handed orthonormal basis $(\vec{u}_r, \vec{u}_\theta)$ in which:



- $\vec{u}_r$ is a unit vector colinear to $\overrightarrow{OM}$: $\overrightarrow{u_r} = \frac{\overrightarrow{OM}}{\|\overrightarrow{OM}\|}$
- $\vec{u}_\theta$ is a unit vector obtained by $\vec{u}_r$, by rotating by $\frac{\pi}{2}$ in the anti-clockwise direction.

Note that, in this case, the unit vectors change when point M moves. We say that the $(\vec{u}_r, \vec{u}_\theta)$ basis is a moving basis.

The position vector, which points to point M, is now written as:

$$\overrightarrow{OM} = r \cdot \vec{u}_r = \begin{pmatrix} r \\ 0 \end{pmatrix}$$

The polar coordinate system is suitable for the study of circular motion.

c. Connection between polar and cartesian coordinate system

The unit vectors of the polar coordinate system can be expressed in terms of those in the cartesian coordinate system as follows:

$$\vec{u}_r = \cos\theta \cdot \vec{u}_x + \sin\theta \cdot \vec{u}_y$$

$$\vec{u}_\theta = -\sin\theta \cdot \vec{u}_x + \cos\theta \cdot \vec{u}_y$$

3. The velocity and acceleration vectors

a. Definitions

The velocity vector is given by the first derivative of the position vector with respect to time:

$$\vec{v}(t) = \frac{d\overrightarrow{OM}}{dt}$$

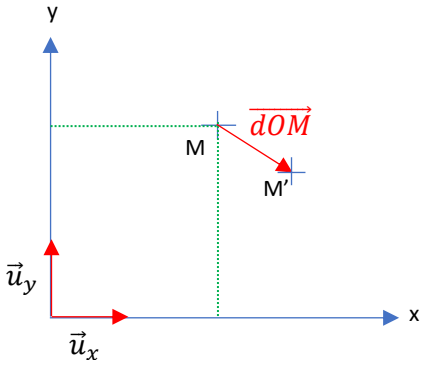
Velocity is colinear to displacement (that is, the position vector). It is therefore parallel to the trajectory. The norm of velocity is measured in meters per second ($m{\cdot}s^{-1}$).

The acceleration vector is given by the first derivative of the velocity vector with respect to time:

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

The norm of acceleration is measured in meters per second ($m{\cdot}s^{-2}$).

b. In cartesian coordinates



As shown in the figure, an infinitesimal displacement $d\overrightarrow{OM}$ can be written as:

$$d\overrightarrow{OM} = dx \cdot \vec{u}_x + dy \cdot \vec{u}_y$$

Therefore, the velocity vector is

$$\vec{v}(t) = \frac{d\overrightarrow{OM}}{dt} = \frac{dx \cdot \vec{u}_x + dy \cdot \vec{u}_y}{dt} = \frac{dx}{dt}\vec{u}_x + \frac{dy}{dt}\vec{u}_y$$
$$= \dot{x}\,\vec{u}_x + \dot{y}\,\vec{u}_y = \begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}$$

Similarly, the acceleration vector is written as:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d\dot{x} \cdot \vec{u}_x + d\dot{y} \cdot \vec{u}_y}{dt} = \frac{d\dot{x}}{dt}\vec{u}_x + \frac{d\dot{y}}{dt}\vec{u}_y = \ddot{x}\,\vec{u}_x + \ddot{y}\,\vec{u}_y = \begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}$$

The sign of the scalar product $\vec{v} \cdot \vec{a}$ gives the direction of $\vec{a}$ with respect to $\vec{v}$. There are two possible effects of acceleration: either modify the norm of the velocity vector, or its direction and consequently the trajectory. If $\vec{v}$ and $\vec{a}$ are collinear, then the trajectory does not change. Moreover,

- if $\vec{v} \cdot \vec{a} > 0$ the 2 vectors point to the same direction: the motion is accelerated.
- if $\vec{v} \cdot \vec{a} = 0$ the 2 vectors are orthogonal to each other: only the trajectory is modified (for $\vec{a} \neq \vec{0}$).
- If $\vec{v} \cdot \vec{a} < 0$ the 2 vectors point to opposite directions: the motion is deaccelerated.

c. <u>In polar coordinates</u>

In the polar coordinate system, when point M moves, r and θ change. A change in θ turns the unit basis by an angle $d\theta$! This makes the calculation of the velocity and acceleration vectors completely different because the derivative of the $\vec{u}_r$ and $\vec{u}_\theta$ vectors is not zero.

We now need to use the expression of the polar unit vectors, $\vec{u}_r$ and $\vec{u}_\theta$, in cartesian coordinates to find their derivative with respect to θ:

$$\frac{d\vec{u}_r}{d\theta} = -\sin\,\theta \cdot \vec{u}_x + \cos\theta \cdot \vec{u}_y = \vec{u}_\theta$$

$$\frac{d\vec{u}_\theta}{d\theta} = -\cos\theta \cdot \vec{u}_x - \sin\theta \cdot \vec{u}_y = -\vec{u}_r$$

Thus, their derivatives with respect to time are

$$\frac{d\vec{u}_r}{dt} = \frac{d\vec{u}_r}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta}\,\vec{u}_\theta$$

$$\frac{d\vec{u}_\theta}{dt} = \frac{d\vec{u}_\theta}{d\theta} \cdot \frac{d\theta}{dt} = -\dot{\theta}\,\vec{u}_r$$

And therefore, we obtain the following expression for the velocity and acceleration vectors:

$$\vec{v}(t) = \frac{d\overrightarrow{OM}}{dt} = \frac{d(r \cdot \vec{u}_r)}{dt} = \frac{dr}{dt} \cdot \vec{u}_r + r \cdot \frac{d\,\vec{u}_r}{dt} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta$$

and

$$\begin{aligned}\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d\left(\dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta\right)}{dt} &= \frac{d\dot{r}}{dt}\vec{u}_r + \dot{r}\frac{d\vec{u}_r}{dt} + \frac{dr}{dt}\,\dot{\theta}\vec{u}_\theta + r\frac{d\dot{\theta}}{dt}\vec{u}_\theta + r\dot{\theta}\frac{d\vec{u}_\theta}{dt} \\ &= \ddot{r}\,\vec{u}_r + \dot{r}\dot{\theta}\,\vec{u}_\theta + \dot{r}\dot{\theta}\,\vec{u}_\theta + r\ddot{\theta}\,\vec{u}_\theta + r\dot{\theta}(-\dot{\theta}\,\vec{u}_r) \\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{u}_r + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\vec{u}_\theta\end{aligned}$$

II. Dynamics

Point dynamics is a branch of physics that studies the causes of motion. It is based on the physical concepts founded by Newton with his 3 laws of motion (i.e. unprovable postulates), that we will study here. Newton, by mathematizing physics and putting it into equations, completely changed its paradigm.

1. Definitions

Definition 4: Mass

Mass m is a measure of a body's ability to resist being set in motion. It is measured in kg and is proportional to the amount of matter in a body.

Definition 5: Force

Force $\vec{F}$ is a vector quantity describing an interaction and its ability to modify motion. It is expressed in Newton (N) (in SI: $kg \cdot m \cdot s^{-2}$)

Definition 6: Momentum

To formulate his laws, Newton introduced a new vector quantity called momentum. It is defined as the product of a body's mass with its velocity $\vec{p} = m\vec{v}$ and is measured $kg \cdot m \cdot s^{-1}$.

2. Newton's first law

Newton's first law, often referred to as the law of inertia, is the following:

In a Galilean reference frame, a body remains in the same state of motion if no force is acting on it.

In other words,

- If the body was at rest, it remains at rest.
- If the body was in motion, it stays in motion, which is uniform and rectilinear (that is, in a straight line).

3. Newton's second law

Newton's second law, sometimes called law of acceleration, is the following:

In a Galilean reference frame, the derivative of a body's momentum with respect to time is equal to the sum of all forces acting on it.

In the context of our course, the mass being always constant, this can be written as

$$\sum_i \vec{F_i} = \frac{d\vec{p}}{dt} = m\vec{a}$$

4. Newton's third law

Newton's third law, or law of action-reaction, is the following:

If a body M_1 exerts a force $\vec{F}_{1\to2}$ on body M_2, then M_2 exerts on M_1 a force $\vec{F}_{2\to1} = -\vec{F}_{1\to2}$.

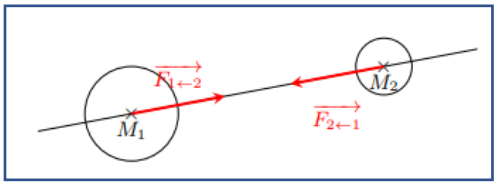
III. Examples of forces

a. Definition

A force is modeled by a vector $\vec{F}$ for which:

- The norm of the vector represents the intensity of the force in Newton (N).
- The direction of the vector represents the direction of the force.
- The origin of the vector is at the point at which the force is applied: it is thus on the object that is subjected to the force.

b. Gravity



The gravitational interaction is a force which leads to the attraction of two bodies with mass towards each other. It is always attractive and has infinite range. Its expression is given by

$$\vec{F}_{1\to2} = G\,\frac{m_1 m_2}{d^2}\,\vec{u}_{2\to1}$$

in which

- G is the universal gravitational constant with value $G = 6.67\cdot10^{-11}\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}$.
- m_1 and m_2 are the masses of the two bodies (measured in kg).
- d is the distance between the two bodies (measured in meters).

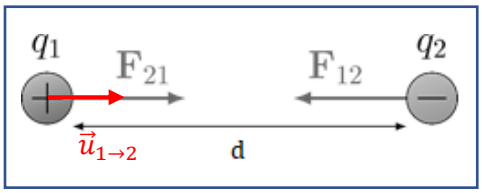
Near the Earth's surface, we can replace the gravitational force by the force that we call weight:

$$\vec{W} = m\vec{g}$$

where g is the gravitational acceleration of the Earth with an approximate value $g = 9.81\ \mathrm{m \cdot s^{-2}}$.

We can obtain this result from the formula for the gravitational force by using the mass of the Earth for m_1 and its radius for d.

c. Electrostatic force



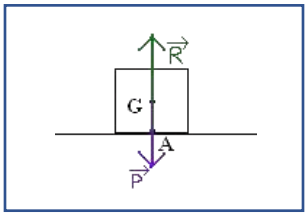
The electrostatic force, known also as the Coulomb force, is a force that acts between two charged particles. Its expression is analogous to the one for gravitational attraction but for electric charges

$$\vec{F}_{1\to 2} = k\frac{q_1 q_2}{d^2}\,\vec{u}_{1\to 2}$$

where k is known as the Coulomb constant with an approximate value $k = 8.88 \cdot 10^9\ \mathrm{N \cdot m^2 \cdot C^{-2}}$.

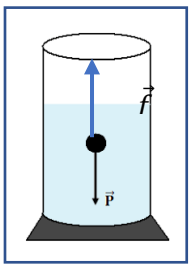
Note that, unlike mass, the electric charge can be negative, and thus the Coulomb force can be attractive or repulsive.

d. Surface reaction force



An object resting on a surface will experience a so-called surface reaction force, denoted $\vec{R}$. It is this force that prevents the object from passing through the surface. There is no general expression for this force, so it must be determined on a case-by-case basis. It is always orthogonal to the surface.

e. Friction

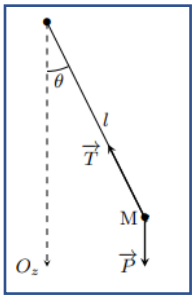


An object moving with a velocity $\vec{v}$ in a fluid will be subjected to a force of friction from the fluid $\vec{f}$:

$$\vec{f} = -\alpha\vec{v}$$

Where α is the friction coefficient and depends on the fluid, the shape of the body, its speed, etc. Note that this force always opposes the motion.

f. Tension in a string

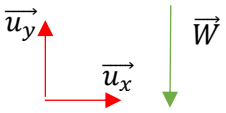


A string attached to an object will exert a force on the latter, for which we cannot establish a general expression. Nevertheless, this force is a contact force, which is applied to the point of attachment of the object, has the direction of the string and points from the object towards the string. The norm must be determined for each problem separately. This force originates from the electromagnetic interaction between the molecules of the string and the object.

IV. Using Newton's second law: an example

When all forces acting on a system are known, Newton's second law allows us to determine the equations of motion. To illustrate this, let's examine the simplest possible system: a material point M of mass m in free fall from an initial height $y_0 = 5\text{m}$, without initial speed.

Example: Free fall



- Reference frame: terrestrial
- System: material point of mass m
- Forces involved: weight

Starting from Newton's second law:

$$\sum_i \vec{F_i} = \frac{d\vec{p}}{dt} = m\vec{a}$$

$$\vec{W} = m\vec{a} \Rightarrow -W = ma \Rightarrow -mg = ma \Rightarrow -g = a$$

Note that the acceleration does not depend on the mass of the object! Using the definition of acceleration we find:

$$v(t) = \int a(t)\ dt = \int -g\ dt = -gt + c_1$$

Here, the initial velocity is zero, thus $c_1 = 0$. Let's now determine the position:

$$y(t) = \int v(t)\, dt = \int -gt\ dt = -\frac{gt^2}{2} + c_2$$

Here the initial height is of 5m: $y(0) = 5$, which gives $c_2 = 5$. Thus,

$$y(t) = -\frac{gt^2}{2} + 5$$

We can now calculate several parameters of the motion, for example the duration of the fall: we want to know when the object hits the ground, i.e. when $y(T) = 0$.

$$y(T) = -\frac{gT^2}{2} + 5 = 0 \Rightarrow T = \sqrt{\frac{10}{g}} = \sqrt{\frac{10}{9{,}81}} \approx 1\ \text{s}$$

To find the velocity of the object when it hits the ground, we use the expression of $v(t)$ for $t = 1\ \text{s}$:

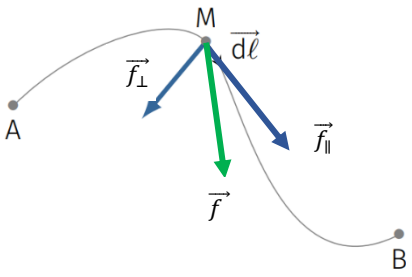
$$v(T) = -gT = -9.81\ \text{m.s}^{-1}$$

V. Energy

Sometimes, we cannot use Newton's laws to derive the equations of motion of a system, especially when the forces involved are too difficult to calculate. In these cases, we use a different approach using the fundamental notion of energy and the principle of energy conservation.

1. Work

Consider a force $\vec{f}$ acting on a system, making it move on a calculable path between points A and B, as shown in the figure. The effect of the force can be better understood if split into two components: one perpendicular to the trajectory, $\overrightarrow{f_\perp}$, and one parallel to it, $\overrightarrow{f_\parallel}$. The first one is responsible for the change in the trajectory, while the second one for the change in speed. To measure how much this parallel component of the force $\overrightarrow{f_\parallel}$ works to accelerate or deaccelerate the system, we define a new physical quantity called work.



Definition 7: Work

Work measures the capacity of a force to change the speed of a system, and describes the energy transferred to/from the system due to the action of the force. It is measured in Joules ($1\text{J} = 1\text{N} \cdot \text{m}$), and is given by the expression:

$$W_{A\to B} = \int_{\mathcal{C}} \vec{f} \cdot \overrightarrow{dl}$$

The vector $\overrightarrow{dl}$ describes the infinitesimal displacement of point M along the trajectory.

Work is given by a line integral that depends, a priori, on both the force and the trajectory. We distinguish three cases:

- $\vec{f} \cdot \overrightarrow{dl} > 0$: the force is driving, resulting in positive work ($W > 0$). The system is accelerated.
- $\vec{f} \cdot \overrightarrow{dl} = 0$: the force is perpendicular to the direction of motion, resulting in no work ($W = 0$). The trajectory is curved without any change in speed.
- $\vec{f} \cdot \overrightarrow{dl} < 0$: the force is resisting, resulting in negative work ($W < 0$). The system slows down.

If the force is constant, then work is given by:

$$W_{A\to B} = \vec{f} \cdot \overrightarrow{AB}$$

2. Power

Power is a quantity that describes how fast a force works, just as electric power describes how fast electric energy is delivered.

Definition 8: Power

Power measures the rate at which energy is transferred to/from an object due to the action of a force $\vec{f}$. It is measured in Watts ($1\text{W} = 1\text{J/s}$) and is given by the expression:

$$P = \frac{dW}{dt} = \vec{f} \cdot \frac{\overrightarrow{dl}}{dt} = \vec{f} \cdot \vec{v}$$

3. The work-kinetic energy theorem

The work-kinetic energy theorem is a theorem that, as its name implies, connects the work done by the forces acting on a system with its kinetic energy. The kinetic energy represents the amount of energy an object has due to its motion. An object with mass m and velocity v has kinetic energy:

$$E_k = \frac{1}{2}mv^2$$

According to the work-kinetic energy theorem, the sum of the works W_i done by all forces acting on a system moving between points A and B is equal to the change of its kinetic energy:

$$\sum_i W_i = \Delta E_k = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

Equivalently, using the notion of power:

$$\sum_i P_i = \frac{dE_k}{dt}$$

Example: Free fall

Let's take as an example the problem of free fall that we have already solved with Newton's second law: a body with mass m falls from a height H, subject only to its own weight and without initial velocity. The work-kinetic energy theorem gives us:

$$\sum_i W_i = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 = mgH$$

Therefore, the velocity of the object when it hits the ground is

$$v = \sqrt{(2gH)}$$

4. Potential energy

A force is called conservative when its work between two points A and B does not depend on the path used, but only on the position of these two points. Such a force is derived by a potential energy E_p which characterizes energetically the object M at each position. Between two points A and B, the work of a conservative force is equal to minus the change in potential energy:

$$W = -\Delta E_p$$

In the case of gravity, E_p is the gravitational potential energy. For motion close to the surface of the Earth, an object with mass m at a height h from the ground has potential energy

$$E_p = mgh$$

with the assumption that $E_p = 0$ on the ground.

5. Conservation of mechanical energy

In all systems in which only conservative forces are acting, the sum of kinetic and potential energy, known as the mechanical energy, is conserved during motion:

$$E_M = E_k + E_p = const.$$

Forces of friction are non-conservative: they dissipate the system's energy.

6. Summary

Definition 9: Energy-related theorems

- The work-kinetic energy theorem:

$$\sum_i W_i = \Delta E_k = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

- The theorem of mechanical energy conservation:

$$E_M = E_k + E_p = const.$$