Physics S2

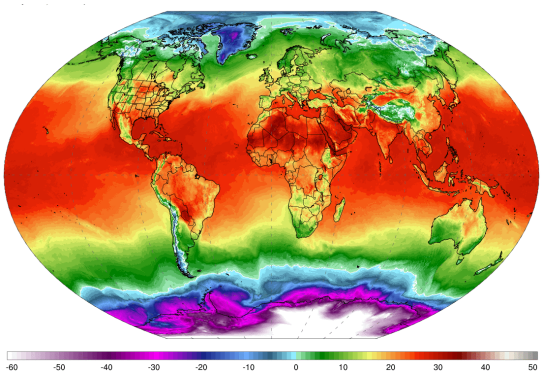
Part 2: Electromagnetism

I. Introduction

Electromagnetism is the branch of physics that describes electric and magnetic phenomena. Even though the Greeks had already discovered materials with electrical properties, such as the amber (elektron in Greek) which has the particularity of attracting small pieces of iron, it was only around 1820 with the work of Oersted that electricity and magnetism became one field. Indeed, he showed that an electric current passing through a wire can move the needle of a compass. Physicists therefore sought to mathematically model how the two branches are connected. Maxwell was the first to propose a list of 20 equations, that he then simplified into a set of 8 equations. Eventually, Heaviside succeeded in condensing them into only 4 equations that are nowadays known as Maxwell's equations.

1. Definitions

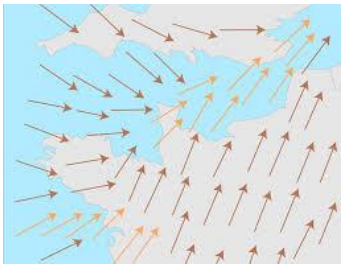
Definition 1: Scalar field



In physics, a field is a quantity defined at every point M in space and may also depend on time. A field is said to be scalar when the associated quantity is a scalar, that is, a number.

For example, the temperature field, defined on a geographical map, is a scalar field: at every point in space, there is an associated number: a temperature.

Definition 2: Vector field



A vector field is a field that associates a vector at every point M in space. This is the case with the wind map.

For every point M, there is a norm and a direction associated.

2. Electric field - Coulomb's law

More than 2600 years ago, the Greeks had already observed that amber rubbed with fur had the ability to attract light bodies such as hair. At the end of the 18th century, the idea that electric charges produce a Newtonian-type force (acting as $1/r^2$) was an attractive hypothesis but difficult to prove experimentally.

It was Coulomb in 1785 who was able to establish a mathematical expression for the force exerted between two charged particles. To do this, he attached a pith ball (B) to a fixed insulating rod and brought another ball (A) close to the first. When the two balls were similarly charged, they moved away, i.e. there was a repulsive force between them. By varying the angle between the two balls and measuring the resulting repulsive force he was able to confirm that the interaction scaled as $1/r^2$.

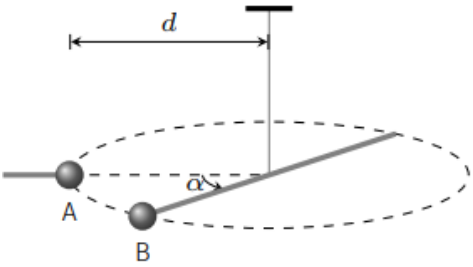


Figure 1: Coulomb's experiment.
(Source : www.femto-physique.fr)

Equation 1: Coulomb's law

Coulomb's law states that the electric force is given by: $\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\vec{u}_{12}$

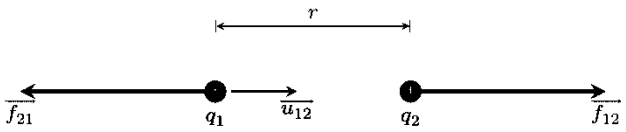


Figure 2: Coulomb's law
(Source : www.femto-physique.fr)

- $k = \frac{1}{4\pi\varepsilon_0} = 8{,}987.10^9$ N m^2 C^{-2}
- q_1 and q_2 are the charges of the two particles (measured in Coulomb, C)
- r is the distance between particles (m)

In electrostatics, we study phenomena in which charges do not move. In such cases, we prefer to use the notion of an electric field: the idea is to look at what would happen if there was only one charge in space, and a second one were then added. It is exactly like in classical mechanics when we talk about gravitational fields. Let's take a small detour!

The gravitational force is given by the expression:

$$\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\vec{u}_{21}$$

Notice the impressive resemblance with the Coulomb force! G here is the universal gravitational constant. When we study the gravitational field of a specific object, for example, the Earth, we can set $m_1 = M_{\text{Earth}}$. For the interaction with objects on the surface of the Earth, we can also set $r = R_{\text{Earth}}$. The gravitational force can thus be rewritten as

$$\vec{F}_{12} = m_2\vec{g} \quad \text{where} \quad \vec{g} = G\frac{m_1}{r^2}\vec{u}_{21}.$$

We can do the same reformulation for the electric force:

$$\vec{F}_{12} = q_2\vec{E} \quad \text{where} \quad \vec{E} = k\frac{q_1}{r^2}\vec{u}_{12}.$$

In the case of the gravitational force, $\vec{g}$ is the gravitational field on the surface of the Earth. In the case of the electric force, $\vec{E}$ describes the electric field created by the charge q_1.

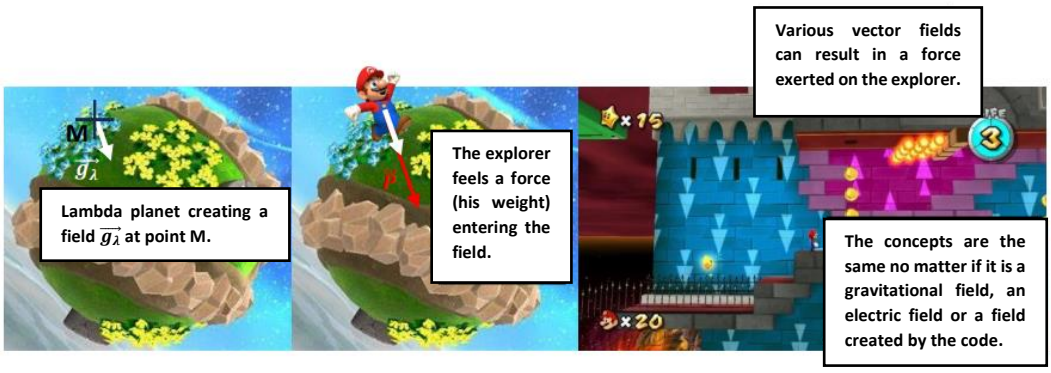


Figure 3 : An example of vector fields.

Since the electric field is a vector field, we must use the proper notation:

- $\overrightarrow{E_1}(M)$ is the electric field vector (measured in N/C or V/m) created at point M by charge 1.
- $\left\|\overrightarrow{E_1}(M)\right\|$ is the norm of the field vector described above.
- E_x, E_y, E_z are the components of the electric field in a cartesian coordinate system.

Note that just like the gravitational field created by the Earth does not affect itself, the electric field created by a charge has no influence on itself. Also note that unlike masses which are always positive, charges can be negative. As a result, the gravitational force is always attractive, but the electric force can be repulsive or attractive.

3. Electric field lines

We can visualize the electric field created by one or more charges by creating a map of the so-called electric field lines:

Definition 3: Field line

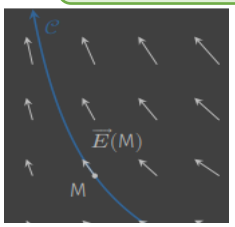


Figure 4 : Electric field lines.
(Source: www.femto-physique.fr)

A field line is a curve oriented along the same direction as the associated vector field. From a mathematical point of view, if $\overrightarrow{dl}$ is an infinitesimal displacement on the field line, then:

$$\overrightarrow{dl} \times \vec{E} = \vec{0}$$

The electric field of a single charge in space is radial, which is consistent with Coulomb's law. The direction of the electric field depends on the sign of the charge: it will be outgoing if the charge is positive, and incoming if it is negative (see Fig.5a, b). When there are two electric charges of opposite sign, the field lines go from the positive charge to the negative charge without ever closing (Fig.5c). On the other hand, if the charges have

the same sign, the field lines of the two charges tend to move away from each other, which illustrates the repulsion between them (Fig.5d).

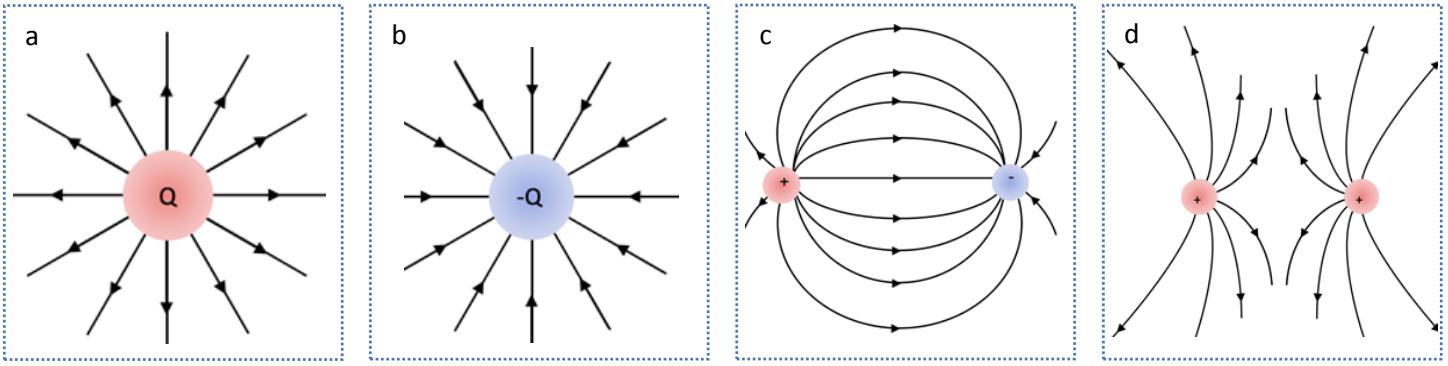


Figure 5: Electric field lines in the case of (a) one positive charge, (b) one negative charge, (c) one positive and one negative charge, and (d) two positive charges. (Source: https://www.physicsclassroom.com/)

II. Charge distributions – Electrostatic energy and potential

1. The principle of superposition

A charge distribution is a collection of electric charges in space. It can be discrete, i.e. the charges are separated from each other, or continuous, i.e. the charges cannot be distinguished.

Every charge in a distribution creates an electric field around it. According to the *principle of superposition*, the electric field resulting from all charges in the distribution is simply the vector sum of the electric field of each charge as if it were alone in space. Thus, to calculate the resulting field, it is sufficient to calculate the field created by each of the charges and then to add them up. We can see this in Figure 5: we can deduce (c) and (d) using the single charge fields (a) and (b).

2. Symmetry and anti-symmetry planes

A charge distribution has a plane of symmetry Π if the charge distribution obtained by symmetry with respect to Π is identical at all points (see Fig. 6a and 6b). A charge distribution has a plane of anti-symmetry Π* if the charge distribution obtained by symmetry with respect to Π* is opposite at all points (see Fig. 6c and 6d).

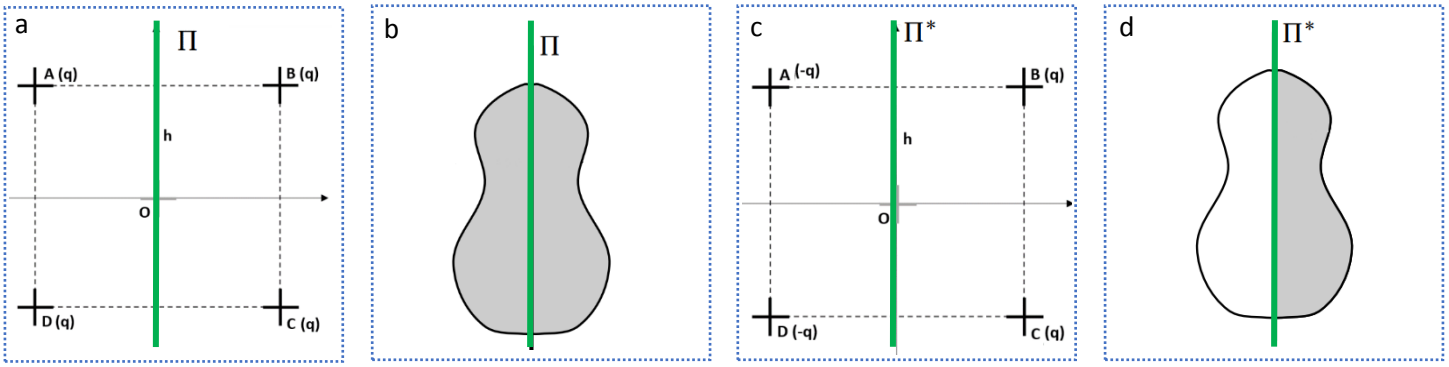
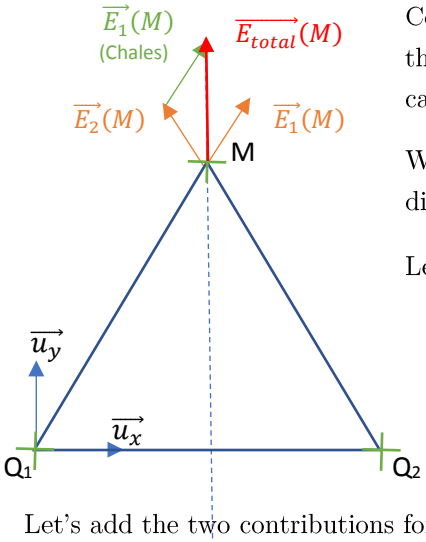


Figure 6 : (a,b) Symmetry and (c,d) anti-symmetry planes.

3. Example

Example 1: The principle of superposition



Consider the case where 2 positive charges $Q_1 = Q_2 = Q$ are placed at the corners of an equilateral triangle with side r. We would like to calculate the resulting field at point M.

We can already notice that there is an axis of symmetry of the charge distribution along the dashed line.

Let's write the fields $\overrightarrow{E_1}$ et $\overrightarrow{E_2}$:

$$\overrightarrow{E_1}(M) = \begin{pmatrix} \frac{kQ}{r^2}\cos(60) \\ \frac{kQ}{r^2}\sin(60) \end{pmatrix} \text{ and } \overrightarrow{E_2}(M) = \begin{pmatrix} -\frac{kQ}{r^2}\cos(60) \\ \frac{kQ}{r^2}\sin(60) \end{pmatrix}$$

Let's add the two contributions for the total field:

$$\overrightarrow{E_{total}}(M) = \overrightarrow{E_1}(M) + \overrightarrow{E_2}(M) = \begin{pmatrix} \frac{kQ}{r^2}\cos(60) - \frac{kQ}{r^2}\cos(60) \\ \frac{kQ}{r^2}\sin(60) + \frac{kQ}{r^2}\sin(60) \end{pmatrix} = \begin{pmatrix} 0 \\ 2\frac{kQ}{r^2}\sin(60) \end{pmatrix}$$

We can notice that the contributions on the x-axis cancel each other out, while the contributions on the y-axis add up. This is consistent with the axis of symmetry. We find the same result by applying the Chales relation with the vectors $\overrightarrow{E_1}(M)$, $\overrightarrow{E_2}(M)$ (see figure).

4. Electrostatic energy

If we have a charge q in space, and we want to bring a second charge q' with the same sign at a distance r, it will cost us energy to do so because of the Coulomb repulsive force. It can be shown that,

Equation 2: Electrostatic energy

The electrostatic energy of two point-charges q and q' at a distance r from each other is given by

$$E_e = k\frac{qq'}{r}$$

(Note that the electrostatic energy scales as 1/r and not as $1/r^2$ like the E field.)

5. Electrostatic potential

In the first section, we defined the electric field $\vec{E}$ around a charge q as a field that illustrates the force $\vec{F}$ that would be exerted on an additional charge q' at distance r:

$$\vec{E} = \frac{\vec{F}}{q'}$$

Similarly, we can define a potential V that illustrates the energy needed for the addition of a charge q' at distance r:

$$V = \frac{E_e}{q'}$$

Equation 3: Electrostatic potential

The electrostatic potential of a point q at a point M located at distance r is given by:

$$V(M) = \frac{kq}{r}$$

It is measured in Volts (V).

The potential is also connected with the electric field through the expression,

$$\vec{E} = -\vec{\nabla} V$$

where $\vec{\nabla}$ is a mathematical operator called gradient. It acts on a scalar function (a scalar field) and returns a vector function (a vector field). Its expression in cartesian coordinates is as follows:

$$\vec{\nabla} f(x,y,z) = \frac{\partial f}{\partial x}\overrightarrow{u_x} + \frac{\partial f}{\partial y}\overrightarrow{u_y} + \frac{\partial f}{\partial z}\overrightarrow{u_z}$$

Thus, in the case of a point charge, $\vec{E} = -\vec{\nabla}\left(\frac{kq}{r}\right) = k\frac{q}{r^2}\vec{u}_r$.

The electrostatic potential has the advantage of being a scalar quantity, easy to manipulate, particularly in the form of a graph. In general, we use equipotential lines in potential maps, i.e. contour lines corresponding to the same potential value, as is done in topographic maps for reliefs. Here is an example for a 2D potential:

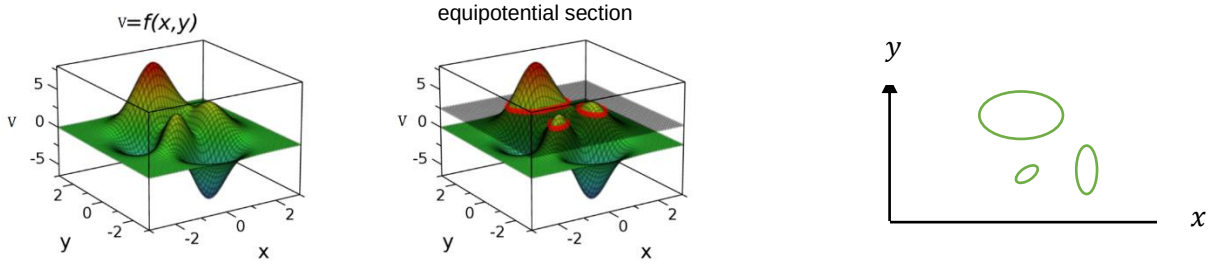


Figure 7 : Representation of a potential (left), an equipotential section at V=2.5Volts (in the middle) and a 2D view (right) of this section, seen from above.

This type of curve is widely used in quantum chemistry where the electric potential of a molecule is represented to better understand its potential interactions with other molecules. See for example the ammonia molecule NH_3 and its ions NH_2^- et NH_4^+ in Fig. 8. We see that the ion NH_4^+ is very slightly polar, i.e. the potential differences are low, that the ammonia molecule NH_3 is rather polar and that the ion NH_2^- is very polar.

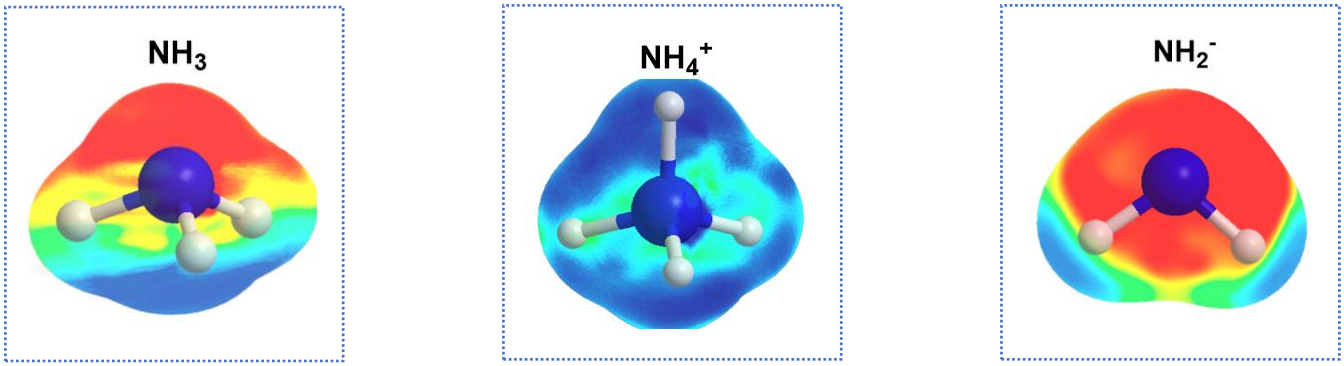


Figure 8 : Representation of the potential for the molecule of ammonia and its ions. (Source: https://www.pearson.com)

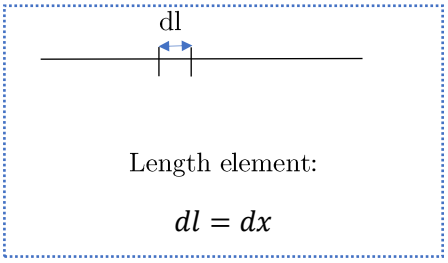
6. Continuous charge distributions

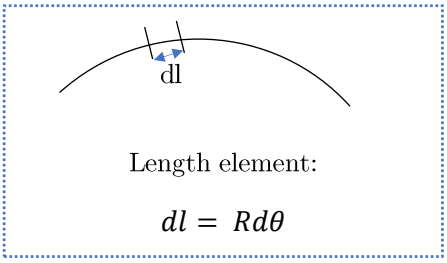
a. Linear charge distributions

In a linear charge distribution, the charges are distributed on a line, e.g. a wire. We call linear charge density λ (measured in C.m^{-1}) the amount of electric charge per meter of wire. For an infinitesimal wire with length dl, there is dQ of electric charge. To calculate the total charge carried by the wire, we must therefore integrate:

$$Q = \int_C \lambda \, dl$$

There are two cases of particular interest:



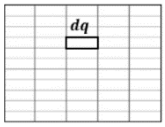


b. Surface charge distributions

In a surface charge distribution, the charges are distributed on a surface. We call surface charge density σ (measured in C.m^{-2}) the amount of electric charge per m^2 of surface. In this case:

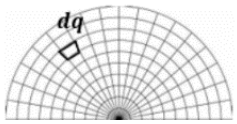
$$Q = \iint \sigma \, dS$$

Again, there are two cases of particular interest:



Surface element:

$$dS = dxdy$$



Surface element:

$$dS = rdrd\theta$$

c. Volume charge distributions

In a volume charge distribution, the charges are distributed in a volume. We call volume charge density ρ (measured in C.m^{-3}) the amount of electric charge per m^3. In this case:

$$Q = \iiint \rho dV$$

III. Electric charges in magnetic fields

1. Introduction

In the presence of a magnetic field, a force known as the Lorentz force is exerted on *moving* electric charges.

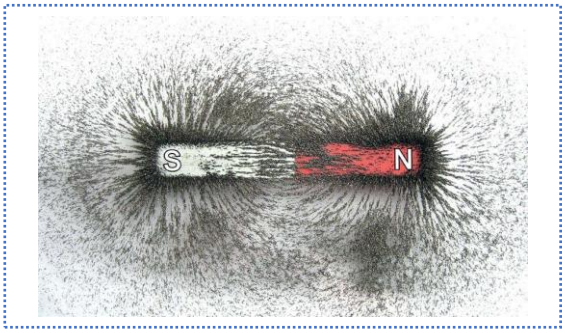
Magnetic fields can be created by

- permanent magnets,
- electric currents i.e. a collective motion of electric charges, or
- time-dependent electric fields.

They are denoted $\vec{B}$ and are measured in Tesla. Here is the order of magnitude of some common fields:

- The magnetic field of the Earth is at 5.10^{-5} T.
- The magnetic field of a fridge magnet is at 10^{-2} T.
- The magnetic field of an MRI scanner is up to 3 T.

We can easily visualize the magnetic field of a magnet by scattering iron filings around it: the filings become magnetized and behave like small compasses that orient themselves according to the local B field.



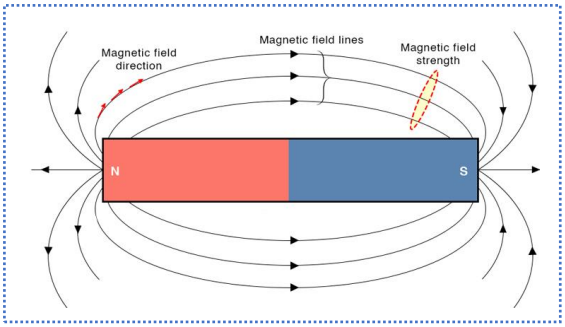


Figure 9: Left: iron filings on a magnet. Right: schematic of the magnetic field. (Source: https://www.magnetmanila.com)

2. Lorentz force

To demonstrate the effect of this force, we can perform the following experiment: we use an electron gun that projects electrons in a uniform rectilinear motion. We apply a magnetic field and observe that the electrons are deflected by the magnetic field. The force responsible for this is called the Lorentz force.

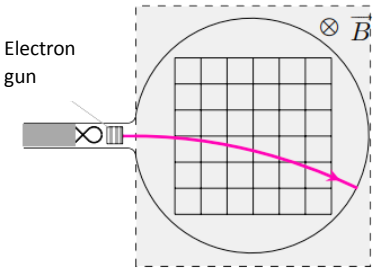


Figure 10: Deviation of the electronic trajectory by a magnetic field.

Formule 4 : Lorentz force

The Lorentz force, which describes the effect of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ on a charged particle q moving with velocity $\vec{v}$, is given by the expression,

$$\vec{F}_{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B}$$

One can recognize in this formula the electrostatic force $\vec{F} = q\vec{E}$.

Also, in the case where the particle is not moving (the velocity is zero), the Lorentz force is limited to the only electrostatic force. The second part of the formula $q\vec{v} \times \vec{B}$ describes the magnetic force (Help on vector products).

This force has the particularity of having zero work. In fact, $\vec{F}_{Lorentz} \perp \vec{v}$: this force does not induce a change in speed. However, this force curves the trajectory of a particle by changing its direction. This property is used in many devices like the mass spectrometers, and cyclotron-type particle accelerators with very broad applications:

- In analytical chemistry: determination of the molecular formula of molecules.
- In environmental chemistry: analysis of air and water; monitoring of pollution from pesticides or industrial processes.
- In biochemistry: identification of proteins (amino acid sequencing) and microorganisms; analysis of blood gases; pharmacology; toxicology.
- In fundamental physics: measuring the mass of stable atoms, creation of high-energy particles.
- In Earth sciences: measurement of isotopic ratios (geology, oceanography, glaciology, volcanology, atmospheric physics, study of meteorites, planetology, etc.).